ΟΡΙΑ ΜΕ ΠΑΡΑΜΕΤΡΟ ΟΤΑΝ $x \to x_0$

Πως θα βρω το $\lim\limits_{x \to x_0} \frac{f(x)}{g(x)}$ όταν στους τύπους των συναρτήσεων f, g εμφανίζονται οι παράμετροι $\lambda, \mu, \ldots$

↓

Διακρίνω τις περιπτώσεις: $\begin{Bmatrix} (\mathrm{I}) \lim\limits_{x \to x_0} f(x) = \lim\limits_{x \to x_0} g(x) = 0 \\ (\mathrm{II}) \lim\limits_{x \to x_0} f(x) \neq 0, \lim\limits_{x \to x_0} g(x) = 0 \\ (\mathrm{III}) \lim\limits_{x \to x_0} g(x) \neq 0 \end{Bmatrix}$

ΠΑΡΑΔΕΙΓΜΑΤΑ

1.

Να βρεθεί το όριο $\lim\limits_{x \to 1} \frac{x^2 - \lambda x + 1}{x^2 + \mu x + 1}$

Θεωρώ την συνάρτηση $f(x) = \frac{x^2 - \lambda x + 1}{x^2 + \mu x + 1}$

Διακρίνω τις περιπτώσεις: $\begin{Bmatrix} (\mathrm{I}) \lim\limits_{x \to 1}(x^2 - \lambda x + 1) = \lim\limits_{x \to 1}(x^2 + \mu x + 1) = 0 \\ (\mathrm{II}) \lim\limits_{x \to 1}(x^2 - \lambda x + 1) \neq 0, \lim\limits_{x \to 1}(x^2 + \mu x + 1) = 0 \\ (\mathrm{III}) \lim\limits_{x \to 1}(x^2 + \mu x + 1) \neq 0 \end{Bmatrix}$

Περίπτωση(I):

$$\begin{Bmatrix} \lim\limits_{x \to 1}(x^2 - \lambda x + 1) = 0 \\ \lim\limits_{x \to 1}(x^2 + \mu x + 1) = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} 2 - \lambda = 0 \\ 2 + \mu = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \lambda = 2 \\ \mu = -2 \end{Bmatrix}$$

Αν $\lambda = 2, \mu = -2$ ο τύπος της συνάρτησης f γίνεται:

$$f(x) = \frac{x^2 - \lambda x + 1}{x^2 + \mu x + 1} \overset{\lambda = 2, \mu = -2}{=} \frac{x^2 - 2x + 1}{x^2 - 2x + 1} = 1$$

Οπότε: $\lim\limits_{x \to 1} f(x) = 1$

Περίπτωση(II):

$$\begin{Bmatrix} \lim\limits_{x \to 1}(x^2 - \lambda x + 1) \neq 0 \\ \lim\limits_{x \to 1}(x^2 + \mu x + 1) = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \lambda \neq 2 \\ \mu = -2 \end{Bmatrix}$$

$A\nu\ \mu=-2\ o\ \tau\acute{\upsilon}\pi o\varsigma\ \tau\eta\varsigma\ \sigma\upsilon\nu\acute{\alpha}\rho\tau\eta\sigma\eta\varsigma\ f\ \gamma\acute{\iota}\nu\varepsilon\tau\alpha\iota:$

$$f(x)=\frac{x^2-\lambda x+1}{x^2+\mu x+1}\overset{\mu=-2}{=}\frac{x^2-\lambda x+1}{x^2-2x+1}=\frac{x^2-\lambda x+1}{x^2-2\bullet x\bullet 1+1^2}=\frac{1}{(x-1)^2}\left(x^2-\lambda x+1\right)$$

$$\lim_{x\to 1}f(x)=\lim_{x\to 1}\frac{1}{(x-1)^2}\left(x^2-\lambda x+1\right)=\lim_{x\to 1}\frac{1}{(x-1)^2}\lim_{x\to 1}\left(x^2-\lambda x+1\right)=$$

$$=(+\infty)(2-\lambda)=\begin{cases}+\infty, 2-\lambda>0\\ -\infty, 2-\lambda<0\end{cases}=\begin{cases}+\infty, -\lambda>-2\\ -\infty, -\lambda<-2\end{cases}=\begin{cases}+\infty, \lambda<2\\ -\infty, \lambda>2\end{cases}$$

$\underline{\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta(\mathrm{III}):}$

$$\lim_{x\to 1}\left(x^2+\mu x+1\right)\neq 0\Leftrightarrow \mu\neq -2$$

$$\lim_{x\to 1}f(x)=\lim_{x\to 1}\frac{x^2-\lambda x+1}{x^2+\mu x+1}=\frac{2-\lambda}{2+\mu}$$

2.

$N\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \acute{o}\rho\iota o\ \lim_{x\to\alpha}\frac{\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}}{\alpha^2-\beta x}, \alpha\beta\neq 0$

$\Theta\varepsilon\omega\rho\acute{\omega}\ \tau\eta\nu\ \sigma\upsilon\nu\acute{\alpha}\rho\tau\eta\sigma\eta\ f(x)\frac{\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}}{\alpha^2-\beta x}$

$$\Delta\iota\alpha\kappa\rho\acute{\iota}\nu\omega\ \tau\iota\varsigma\ \pi\varepsilon\rho\iota\pi\tau\acute{\omega}\sigma\varepsilon\iota\varsigma:\left\{\begin{matrix}(\mathrm{I})\lim_{x\to\alpha}\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)=\lim_{x\to\alpha}\left(\alpha^2-\beta x\right)=0\\ (\mathrm{II})\lim_{x\to\alpha}\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)\neq 0, \lim_{x\to\alpha}\left(\alpha^2-\beta x\right)=0\\ (\mathrm{III})\lim_{x\to\alpha}\left(\alpha^2-\beta x\right)\neq 0\end{matrix}\right\}$$

$\underline{\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta(\mathrm{I}):}$

$$\left\{\begin{matrix}\lim_{x\to\alpha}\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)=0\\ \lim_{x\to\alpha}\left(\alpha^2-\beta x\right)=0\\ \alpha\neq 0, \beta\neq 0\end{matrix}\right\}\Leftrightarrow\left\{\begin{matrix}\sqrt{2\alpha^2}-\alpha\sqrt{2}=0\\ \alpha^2-\alpha\beta=0\\ \alpha\neq 0, \beta\neq 0\end{matrix}\right\}\Leftrightarrow\left\{\begin{matrix}2\left(\sqrt{\alpha^2}-\alpha\right)=0\\ \alpha^2-\alpha\beta=0\\ \alpha\neq 0, \beta\neq 0\end{matrix}\right\}\overset{\sqrt{\alpha^2}=|\alpha|}{\Leftrightarrow}$$

$$\Leftrightarrow\left\{\begin{matrix}|\alpha|-\alpha=0\\ \alpha(\alpha-\beta)=0\\ \alpha\neq 0, \beta\neq 0\end{matrix}\right\}\Leftrightarrow\left\{\begin{matrix}|\alpha|=\alpha\\ \alpha-\beta=0\\ \alpha\neq 0, \beta\neq 0\end{matrix}\right\}\overset{|x|=x\Leftrightarrow x\geq 0}{\Leftrightarrow}\left\{\begin{matrix}\alpha\geq 0\\ \beta=\alpha\\ \alpha\neq 0, \beta\neq 0\end{matrix}\right\}\Leftrightarrow\left\{\begin{matrix}\alpha>0\\ \beta=\alpha\end{matrix}\right\}$$

Αν $\beta=\alpha, \alpha>0$ ο τύπος της f γίνεται:

$$f(x)=\frac{\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}}{\alpha^2-\beta x}=\frac{\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}}{\alpha^2-\alpha x}=\frac{\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)\left(\sqrt{x^2+\alpha^2}+\alpha\sqrt{2}\right)}{\alpha(\alpha-x)\left(\sqrt{x^2+\alpha^2}+\alpha\sqrt{2}\right)}$$

$$=\frac{\left(\sqrt{x^2+\alpha^2}\right)^2-\left(\alpha\sqrt{2}\right)^2}{\alpha(\alpha-x)\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)}=\frac{x^2+\alpha^2-2\alpha^2}{\alpha(\alpha-x)\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)}=$$

$$\frac{x^2-\alpha^2}{\alpha\left[-(x-\alpha)\right]\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)}=-\frac{\cancel{(x-\alpha)}(x+\alpha)}{\alpha\cancel{(x-\alpha)}\left(\sqrt{x^2+\alpha^2}+\alpha\sqrt{2}\right)}=$$

$$-\frac{x+\alpha}{\alpha\left(\sqrt{x^2+\alpha^2}+\alpha\sqrt{2}\right)}$$

$$\lim_{x\to a} f(x)=-\lim_{x\to a}\frac{x+\alpha}{\alpha\left(\sqrt{x^2+\alpha^2}+\alpha\sqrt{2}\right)}=-\frac{2\cancel{a}}{\cancel{\alpha}\left(\sqrt{2a^2}+\alpha\sqrt{2}\right)}=$$

$$-\frac{2}{\sqrt{2}\sqrt{a^2}+\alpha\sqrt{2}}=-\frac{2}{\sqrt{2}|a|+\alpha\sqrt{2}}\overset{a>0\Rightarrow|a|=a}{=}-\frac{2}{\sqrt{2}a+\alpha\sqrt{2}}=-\frac{\cancel{2}}{\cancel{2}\sqrt{2}a}=-\frac{\sqrt{2}}{\sqrt{2}\sqrt{2}a}=$$

$$=-\frac{\sqrt{2}}{2a}$$

<u>Περίπτωση (ΙΙ):</u>

$$\left\{\begin{array}{l}\lim\limits_{x\to\alpha}\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)\neq 0\\ \lim\limits_{x\to\alpha}\left(\alpha^2-\beta x\right)=0\\ \alpha\neq 0, \beta\neq 0\end{array}\right\}\Leftrightarrow\left\{\begin{array}{l}\alpha<0\\ \beta=\alpha\end{array}\right\}$$

Αν $\beta=\alpha$ ο τύπος της f γίνεται:

$$f(x)=\frac{\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}}{\alpha^2-\beta x}\overset{\beta=\alpha}{=}\frac{\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}}{\alpha^2-\alpha x}=\frac{\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}}{-\alpha(x-a)}=$$

$$=-\frac{1}{\alpha}\frac{1}{x-a}\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)$$

$$\lim_{x\to a^-} f(x)=-\frac{1}{\alpha}\lim_{x\to a^-}\frac{1}{x-a}\lim_{x\to a^-}\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right)=-\frac{1}{\alpha}(-\infty)\left(\sqrt{2\alpha^2}-\alpha\sqrt{2}\right)$$

$$\overset{\alpha<0\Rightarrow -\frac{1}{\alpha}>0}{=} (-\infty)\left(\sqrt{2}\sqrt{\alpha^2}-\alpha\sqrt{2}\right) \overset{\sqrt{\alpha^2}=|\alpha|}{=} (-\infty)\left(\sqrt{2}|\alpha|-\alpha\sqrt{2}\right) \overset{\alpha<0\Rightarrow|\alpha|=-\alpha}{=}$$

$$(-\infty)\left(-\sqrt{2}\alpha-\alpha\sqrt{2}\right)=(-\infty)\left(-2\sqrt{2}\alpha\right) \overset{\alpha<0\Rightarrow -2\sqrt{2}\alpha>0}{=} -\infty$$

$$\lim_{x\to a^+} f(x) = -\frac{1}{\alpha}\lim_{x\to a^-}\frac{1}{x-a}\lim_{x\to a^-}\left(\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}\right) = -\frac{1}{\alpha}(+\infty)\left(\sqrt{2\alpha^2}-\alpha\sqrt{2}\right)$$

$$\overset{\alpha<0\Rightarrow -\frac{1}{\alpha}>0}{=} (+\infty)\left(\sqrt{2}\sqrt{\alpha^2}-\alpha\sqrt{2}\right) \overset{\sqrt{\alpha^2}=|\alpha|}{=} (+\infty)\left(\sqrt{2}|\alpha|-\alpha\sqrt{2}\right) \overset{\alpha<0\Rightarrow|\alpha|=-\alpha}{=}$$

$$(+\infty)\left(-\sqrt{2}\alpha-\alpha\sqrt{2}\right)=(+\infty)\left(-2\sqrt{2}\alpha\right) \overset{\alpha<0\Rightarrow -2\sqrt{2}\alpha>0}{=} +\infty$$

Επειδή $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ *δεν υπάρχει το όριο της συνάρτησης* f *στο σημείο* $x_0=\alpha$

<u>Περίπτωση (III)</u>:

$$\left\{\begin{matrix}\lim_{x\to\alpha}\left(\alpha^2-\beta x\right)=0\\ \alpha\neq 0,\beta\neq 0\end{matrix}\right\} \Leftrightarrow 0\neq\beta\neq\alpha\neq 0$$

$$\lim_{x\to a} f(x) = \lim_{x\to a}\frac{\sqrt{x^2+\alpha^2}-\alpha\sqrt{2}}{\alpha^2-\beta x} = \frac{\sqrt{2\alpha^2}-\alpha\sqrt{2}}{\alpha^2-\beta\alpha} = \frac{\sqrt{2}\sqrt{\alpha^2}-\alpha\sqrt{2}}{\alpha(\alpha-\beta)} \overset{\sqrt{\alpha^2}=|\alpha|}{=}$$

$$\frac{\sqrt{2}|\alpha|-\alpha\sqrt{2}}{\alpha(\alpha-\beta)} = \begin{cases}\dfrac{\sqrt{2}|\alpha|-\alpha\sqrt{2}}{\alpha(\alpha-\beta)},\alpha>0\\ \dfrac{\sqrt{2}|\alpha|-\alpha\sqrt{2}}{\alpha(\alpha-\beta)},\alpha<0\end{cases} = \begin{cases}\dfrac{\sqrt{2}\alpha-\alpha\sqrt{2}}{\alpha(\alpha-\beta)},\alpha>0\\ \dfrac{\sqrt{2}(-\alpha)-\alpha\sqrt{2}}{\alpha(\alpha-\beta)},\alpha<0\end{cases} =$$

$$= \begin{cases}0,\alpha>0\\ \dfrac{-\sqrt{2}\alpha-\alpha\sqrt{2}}{\alpha(\alpha-\beta)},\alpha<0\end{cases} = \begin{cases}0,\alpha>0\\ \dfrac{-2\sqrt{2}\cancel{\alpha}}{\cancel{\alpha}(\alpha-\beta)},\alpha<0\end{cases} = \begin{cases}0,\alpha>0\\ \dfrac{-2\sqrt{2}}{-(\beta-\alpha)},\alpha<0\end{cases} = \begin{cases}0,\alpha>0\\ \dfrac{2\sqrt{2}}{\beta-\alpha},\alpha<0\end{cases}$$

3.

Nα *βρεθεί το όριο* $\lim_{x\to 0}\frac{\sqrt{x^2+\alpha^2}-\alpha}{\sqrt{x^2+\beta^2}-\beta}, \alpha\beta\neq 0$

Θεωρώ την συνάρτηση $f(x)=\frac{\sqrt{x^2+\alpha^2}-\alpha}{\sqrt{x^2+\beta^2}-\beta}$

$$\Delta\iota\alpha\kappa\rho\acute{\iota}\nu\omega\ \tau\iota\varsigma\ \pi\varepsilon\rho\iota\pi\tau\acute{\omega}\sigma\varepsilon\iota\varsigma: \left\{\begin{array}{l} (\mathrm{I})\lim\limits_{x\to 0}\left(\sqrt{x^2+\alpha^2}-\alpha\right)=\lim\limits_{x\to 0}\left(\sqrt{x^2+\beta^2}-\beta\right)=0 \\ (\mathrm{II})\lim\limits_{x\to 0}\left(\sqrt{x^2+\alpha^2}-\alpha\right)\neq 0, \lim\limits_{x\to 0}\left(\sqrt{x^2+\beta^2}-\beta\right)=0 \\ (\mathrm{III})\lim\limits_{x\to 0}\left(\sqrt{x^2+\beta^2}-\beta\right)\neq 0 \end{array}\right\}$$

<u>$\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta\ (\mathrm{I})$:</u>

$$\left\{\begin{array}{l} \lim\limits_{x\to 0}\left(\sqrt{x^2+\alpha^2}-\alpha\right)=0 \\ \lim\limits_{x\to 0}\left(\sqrt{x^2+\beta^2}-\beta\right)=0 \\ \alpha\neq 0,\beta\neq 0 \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \sqrt{\alpha^2}=\alpha \\ \sqrt{\beta^2}=\beta \\ \alpha\neq 0,\beta\neq 0 \end{array}\right\} \overset{\sqrt{x^2}=|x|}{\Longleftrightarrow} \left\{\begin{array}{l} |\alpha|=\alpha \\ |\beta|=\beta \\ \alpha\neq 0,\beta\neq 0 \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \alpha\geq 0 \\ \beta\geq 0 \\ \alpha\neq 0,\beta\neq 0 \end{array}\right\}$$

$$\Leftrightarrow \left\{\begin{array}{l} \alpha>0 \\ \beta>0 \end{array}\right\}$$

$$f(x)=\frac{\sqrt{x^2+\alpha^2}-\alpha}{\sqrt{x^2+\beta^2}-\beta}=\frac{\left(\sqrt{x^2+\alpha^2}-\alpha\right)\left(\sqrt{x^2+\alpha^2}+\alpha\right)\left(\sqrt{x^2+\beta^2}+\beta\right)}{\left(\sqrt{x^2+\beta^2}-\beta\right)\left(\sqrt{x^2+\beta^2}+\beta\right)\left(\sqrt{x^2+\alpha^2}+\alpha\right)}=$$

$$\frac{\left[\left(\sqrt{x^2+\alpha^2}\right)^2-\alpha^2\right]\left(\sqrt{x^2+\beta^2}+\beta\right)}{\left[\left(\sqrt{x^2+\beta^2}\right)^2-\beta^2\right]\left(\sqrt{x^2+\alpha^2}+\alpha\right)}=\frac{\cancel{x^2}\left(\sqrt{x^2+\beta^2}+\beta\right)}{\cancel{x^2}\left(\sqrt{x^2+\alpha^2}+\alpha\right)}=\frac{\sqrt{x^2+\beta^2}+\beta}{\sqrt{x^2+\alpha^2}+\alpha}$$

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{\sqrt{x^2+\beta^2}+\beta}{\sqrt{x^2+\alpha^2}+\alpha}=\frac{\sqrt{\beta^2}+\beta}{\sqrt{\alpha^2}+\alpha}=\frac{|\beta|+\beta}{|\alpha|+\alpha}\overset{\substack{\beta>0\Rightarrow|\beta|=\beta \\ \alpha>0\Rightarrow|\alpha|=\alpha}}{=}\frac{\beta+\beta}{\alpha+\alpha}=\frac{2\beta}{2\alpha}=\frac{\beta}{\alpha}$$

<u>$\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta\ (\mathrm{II})$:</u>

$$\left\{\begin{array}{l} \lim\limits_{x\to 0}\left(\sqrt{x^2+\alpha^2}-\alpha\right)\neq 0 \\ \lim\limits_{x\to 0}\left(\sqrt{x^2+\beta^2}-\beta\right)=0 \\ \alpha\neq 0,\beta\neq 0 \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \alpha<0 \\ \beta>0 \end{array}\right\}$$

$\mathrm{A}\nu\ x\neq 0\ \theta\alpha\ \acute{\varepsilon}\chi\omega$:

$$x\neq 0\Rightarrow x^2>0\Rightarrow x^2+\beta^2>\beta^2\Rightarrow\sqrt{x^2+\beta^2}>\sqrt{\beta^2}\Rightarrow\sqrt{x^2+\beta^2}>|\beta|\overset{\beta>0\Rightarrow|\beta|=\beta}{\Longrightarrow}$$

$$\sqrt{x^2+\beta^2}>\beta\Rightarrow\sqrt{x^2+\beta^2}-\beta>0$$

$$Έχω:\begin{cases}(\mathrm{I})\lim\limits_{x\to 0}\left(\sqrt{x^2+\beta^2}-\beta\right)=0\\(\mathrm{II})\sqrt{x^2+\beta^2}-\beta>0, x\neq 0\end{cases}$$

$$Οπότε:\lim_{x\to 0}\frac{1}{\sqrt{x^2+\beta^2}-\beta}=+\infty$$

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{1}{\sqrt{x^2+\beta^2}-\beta}\lim_{x\to 0}\left(\sqrt{x^2+\alpha^2}-\alpha\right)=(+\infty)\left(\sqrt{\alpha^2}-\alpha\right)=$$

$$(+\infty)(|\alpha|-\alpha)\overset{\alpha<0\Rightarrow|\alpha|=-\alpha}{=}(+\infty)(-\alpha-\alpha)=(+\infty)(-2\alpha)\overset{\alpha<0\Rightarrow -2\alpha>0}{=}+\infty$$

<u>$Περίπτωση\ (\mathrm{III})$:</u>

$$\begin{cases}\lim\limits_{x\to 0}\left(\sqrt{x^2+\beta^2}-\beta\right)\neq 0\\ \alpha\neq 0,\beta\neq 0\end{cases}\Leftrightarrow\begin{cases}\beta<0\\ \alpha\neq 0\end{cases}$$

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{\sqrt{x^2+\alpha^2}-\alpha}{\sqrt{x^2+\beta^2}-\beta}=\frac{\sqrt{\alpha^2}-\alpha}{\sqrt{\beta^2}-\beta}=\frac{|\alpha|-\alpha}{|\beta|-\beta}\overset{\beta<0\Rightarrow|\beta|=-\beta}{=}\frac{|\alpha|-\alpha}{-\beta-\beta}=$$

$$-\frac{|\alpha|-\alpha}{2\beta}=\begin{cases}-\dfrac{|\alpha|-\alpha}{2\beta},\alpha>0\\-\dfrac{|\alpha|-\alpha}{2\beta},\alpha<0\end{cases}=\begin{cases}-\dfrac{\alpha-\alpha}{2\beta},\alpha>0\\-\dfrac{-\alpha-\alpha}{2\beta},\alpha<0\end{cases}=\begin{cases}0,\alpha>0\\-\dfrac{-2\alpha}{2\beta},\alpha<0\end{cases}=\begin{cases}0,\alpha>0\\\dfrac{\alpha}{\beta},\alpha<0\end{cases}$$

<u>ΑΣΚΗΣΕΙΣ</u>

1.

Να βρεθεί το όριο $\lim\limits_{x\to 2}\frac{2x-\lambda^2}{x^2-4x+4}$

2.

Να βρεθεί το όριο $\lim\limits_{x\to \alpha}\frac{\sqrt{x^2+3\alpha^2}-2\alpha}{2\alpha^2-\beta x},\alpha\beta\neq 0$

3.

Να βρεθεί το όριο $\lim\limits_{x\to 0}\frac{\sqrt{x^2+9\alpha^2}-3\alpha}{\sqrt{x^2+4\beta^2}-2\beta},\alpha\beta\neq 0$

4.

Να βρεθεί το όριο $\lim\limits_{x\to 1}\frac{2\lambda x^2+\lambda^2 x-3}{x+\sqrt{x}-x\sqrt{x}-1}$