ΕΥΡΕΣΗ ΟΡΙΟΥ ΣΥΝΑΡΤΗΣΗΣ ΟΤΑΝ $x \to \pm\infty$ (Νο2)

ΠΑΡΑΔΕΙΓΜΑΤΑ

1.

Δίνεται η συνάρτηση $f(x)=\sqrt[3]{8x^3+3x+1}-2x$. *Να βρεθεί το όριο* $\lim\limits_{x\to+\infty} f(x)$

$$f(x)=\sqrt[3]{8x^3+3x+1}-2x=\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}-2x=\sqrt[3]{x^3}\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}-2x$$

$$\overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=} x\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}-2x=x\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}-2\right)$$

Έχω:

$$\lim_{x\to+\infty} x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}-2\right)=\sqrt[3]{8+\lim_{x\to+\infty}\frac{3}{x^2}+\lim_{x\to+\infty}\frac{1}{x^3}}-2=\sqrt[3]{8+0+0}-2=2-2=0$$

$$f(x)=\sqrt[3]{8x^3+3x+1}-2x=$$

$$=\frac{\left(\sqrt[3]{8x^3+3x+1}-2x\right)\left[\left(\sqrt[3]{8x^3+3x+1}\right)^2+\sqrt[3]{8x^3+3x+1}\bullet 2x+(2x)^2\right]}{\left(\sqrt[3]{8x^3+3x+1}\right)^2+\sqrt[3]{8x^3+3x+1}\bullet 2x+(2x)^2}\overset{(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)=\alpha^3-\beta^3}{=}$$

$$=\frac{\left(\sqrt[3]{8x^3+3x+1}\right)^3-(2x)^3}{\left(\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}\right)^2+2x\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}+4x^2}=$$

$$=\frac{8x^3+3x+1-8x^3}{\left(\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}\right)^2+2x\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}+4x^2}=$$

$$=\frac{8x^3+3x+1-8x^3}{\left(\sqrt[3]{x^3}\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2x\sqrt[3]{x^3}\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4x^2}\overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=}$$

$$=\frac{x\left(3+\frac{1}{x}\right)}{\left(x\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2x\bullet x\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4x^2}=$$

$$=\frac{x\left(3+\frac{1}{x}\right)}{x^2\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2x^2\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4x^2}=$$

$$=\frac{x\left(3+\frac{1}{x}\right)}{x^2\left[\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}x+4\right]}=$$

$$=\frac{1}{x}\frac{3+\frac{1}{x}}{\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4}$$

$$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}\frac{1}{x}\frac{3+\lim_{x\to+\infty}\frac{1}{x}}{\left(\sqrt[3]{8+\lim_{x\to+\infty}\frac{3}{x^2}+\lim_{x\to+\infty}\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\lim_{x\to+\infty}\frac{3}{x^2}+\frac{1}{x^3}}+4}=$$

$$=0\bullet\frac{3+0}{\left(\sqrt[3]{8+0+0}\right)^2+2\sqrt[3]{8+0+0}+4}=\frac{3}{3\bullet 4}=\frac{1}{4}$$

2.

$\Delta\acute{\iota}\nu\varepsilon\tau\alpha\iota\ \eta\ \sigma\upsilon\nu\acute{\alpha}\rho\tau\eta\sigma\eta\ f(x)=\frac{\sqrt[3]{x^3+3x+1}-x}{x^2+1}$.Να βρεθεί το όριο $\lim_{x\to+\infty}f(x)$

$$\sqrt[3]{x^3+3x+1}-x=\sqrt[3]{x^3\left(1+\frac{3}{x^2}+\frac{1}{x^3}\right)}-x=\sqrt[3]{x^3}\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}-x\overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=}$$

$$x\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}-x=x\left(\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}-1\right)$$

Έχω:

$$\lim_{x\to+\infty}x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}-1\right)=\sqrt[3]{1+\lim_{x\to+\infty}\frac{3}{x^2}+\lim_{x\to+\infty}\frac{1}{x^3}}-1=\sqrt[3]{1+0+0}-1=1-1=0$$

$$f(x)=\frac{\sqrt[3]{x^3+3x+1}-x}{x^2+1}=\frac{\left(\sqrt[3]{x^3+3x+1}-x\right)\left[\left(\sqrt[3]{x^3+3x+1}\right)^2+\sqrt[3]{x^3+3x+1}\bullet x+x^2\right]}{x^2\left(1+\frac{1}{x^2}\right)\left[\left(\sqrt[3]{x^3+3x+1}\right)^2+\sqrt[3]{x^3+3x+1}\bullet x+x^2\right]}$$

$$\overset{(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)=\alpha^3-\beta^3}{=} \frac{\left(\sqrt[3]{x^3+3x+1}\right)^3 - x^3}{x^2\left(1+\frac{1}{x^2}\right)\left[\left(\sqrt[3]{x^3\left(1+\frac{3}{x^2}+\frac{1}{x^3}\right)}\right)^2 + x\sqrt[3]{x^3\left(1+\frac{3}{x^2}+\frac{1}{x^3}\right)} + x^2\right]} =$$

$$\frac{x^3+3x+1-x^3}{x^2\left(1+\frac{1}{x^2}\right)\left[\left(\sqrt[3]{x^3}\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2 + x\sqrt[3]{x^3}\sqrt[3]{x^3\left(1+\frac{3}{x^2}+\frac{1}{x^3}\right)} + x^2\right]} \overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=}$$

$$\frac{x\left(3+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x^2}\right)\left[\left(x\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2 + x\bullet x\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}} + x^2\right]} =$$

$$\frac{x\left(3+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x^2}\right)\left[x^2\left(\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2 + x^2\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}} + x^2\right]} =$$

$$\frac{x\left(3+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x^2}\right)x^2\left[\left(\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2 + \sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}} + 1\right]} =$$

$$\frac{x\left(3+\frac{1}{x}\right)}{x^4\left(1+\frac{1}{x^2}\right)\left[\left(\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2 + \sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}} + 1\right]} =$$

$$\frac{1}{x^3}\frac{3+\frac{1}{x}}{\left(1+\frac{1}{x^2}\right)\left[\left(\sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2 + \sqrt[3]{1+\frac{3}{x^2}+\frac{1}{x^3}} + 1\right]}$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}\frac{1}{x^3}\frac{3+\lim\limits_{x\to+\infty}\frac{1}{x}}{\left(1+\lim\limits_{x\to+\infty}\frac{1}{x^2}\right)\left[\left(\sqrt[3]{1+\lim\limits_{x\to+\infty}\frac{3}{x^2}+\lim\limits_{x\to+\infty}\frac{1}{x^3}}\right)^2 + \sqrt[3]{x^3\left(1+\frac{3}{x^2}+\frac{1}{x^3}\right)} + 1\right]} =$$

$$= 0\,\frac{3+0}{(1+0)\left[\left(\sqrt[3]{1+0+0}\right)^2 + \sqrt[3]{1+0+0} + 1\right]} = 0$$

3.

$$\Delta\acute{\iota}\nu\varepsilon\tau\alpha\iota\ \eta\ \sigma\upsilon\nu\acute{\alpha}\rho\tau\eta\sigma\eta\ f(x)=\frac{\sqrt[3]{8x^3+5x+1}-2x}{\sqrt{x^2+1}-x}.\text{N}\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \acute{o}\rho\iota o\ \lim_{x\to+\infty} f(x)$$

$$\sqrt[3]{8x^3+5x+1}-2x=\sqrt[3]{x^3\left(8+\frac{5}{x^2}+\frac{1}{x^3}\right)}-2x=\sqrt[3]{x^3}\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}-2x\overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=}$$

$$x\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}-x=x\left(\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}-2\right)$$

$\acute{E}\chi\omega$:

$$\lim_{x\to+\infty} x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}-2\right)=\sqrt[3]{8+\lim_{x\to+\infty}\frac{5}{x^2}+\lim_{x\to+\infty}\frac{1}{x^3}}-2=\sqrt[3]{8+0+0}-2=2-2=0$$

$$\sqrt{x^2+1}-x=\sqrt{x^2\left(1+\frac{1}{x^2}\right)}-x=\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}-x\overset{\sqrt{x^2}=|x|}{=}|x|\sqrt{1+\frac{1}{x^2}}-x\overset{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x}{=}$$

$$x\sqrt{1+\frac{1}{x^2}}-x=x\left(\sqrt{1+\frac{1}{x^2}}-1\right)$$

$\acute{E}\chi\omega$:

$$\lim_{x\to+\infty} x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt{1+\frac{1}{x^2}}-1\right)=\sqrt{1+\lim_{x\to+\infty}\frac{1}{x^2}}-1=\sqrt{1+0}-1=1-1=0$$

$$f(x)=\frac{\sqrt[3]{8x^3+5x+1}-2x}{\sqrt{x^2+1}-x}=$$

$$\frac{\left(\sqrt[3]{8x^3+5x+1}-2x\right)\left[\left(\sqrt[3]{8x^3+5x+1}\right)^2+\sqrt[3]{8x^3+5x+1}\bullet 2x+(2x)^2\right]\left(\sqrt{x^2+1}+x\right)}{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)\left[\left(\sqrt[3]{8x^3+5x+1}\right)^2+\sqrt[3]{8x^3+5x+1}\bullet 2x+(2x)^2\right]}=$$

$$\frac{\left[\left(\sqrt[3]{8x^3+5x+1}\right)^3-(2x)^3\right]\left(\sqrt{x^2+1}+x\right)}{\left[\left(\sqrt{x^2+1}\right)^2-x^2\right]\left[\left(\sqrt[3]{8x^3+5x+1}\right)^2+2x\sqrt[3]{8x^3+5x+1}+(2x)^2\right]}=$$

$$\frac{\left(8x^3+5x+1-8x^3\right)\left[\sqrt{x^2\left(1+\frac{1}{x^2}\right)}+x\right]}{\left(x^2+1-x^2\right)\left[\left(\sqrt[3]{x^3\left(8+\frac{5}{x^2}+\frac{1}{x^3}\right)}\right)^2+2x\sqrt[3]{x^3\left(8+\frac{5}{x^2}+\frac{1}{x^3}\right)}+4x^2\right]}$$

$$\frac{x\left(5+\frac{1}{x}\right)\left[\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}+x\right]}{\left(x^2+1-x^2\right)\left[\left(\sqrt[3]{x^3\left(8+\frac{5}{x^2}+\frac{1}{x^3}\right)}\right)^2+2x\sqrt[3]{x^3\left(8+\frac{5}{x^2}+\frac{1}{x^3}\right)}+4x^2\right]}\overset{\sqrt{x^2}=|x|}{=}$$

$$\frac{x\left(5+\frac{1}{x}\right)\left[|x|\sqrt{1+\frac{1}{x^2}}+x\right]}{\left(\sqrt[3]{x^3}\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}\right)^2+2x\sqrt[3]{x^3}\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}+4x^2}\overset{\substack{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x\\ x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}}{=}$$

$$\frac{x\left(5+\frac{1}{x}\right)\left(x\sqrt{1+\frac{1}{x^2}}+x\right)}{\left[\left(x\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}\right)^2+2x\bullet x\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}+4x^2\right]}=$$

$$\frac{x\left(5+\frac{1}{x}\right)x\left(\sqrt{1+\frac{1}{x^2}}+1\right)}{x^2\left(\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}\right)^2+2x^2\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}+4x^2}=$$

$$\frac{\cancel{x^2}\left(5+\frac{1}{x}\right)\left(\sqrt{1+\frac{1}{x^2}}+1\right)}{\cancel{x^2}\left[\left(\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}+4\right]}=\frac{\left(5+\frac{1}{x}\right)\left(\sqrt{1+\frac{1}{x^2}}+1\right)}{\left(\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\frac{5}{x^2}+\frac{1}{x^3}}+4}$$

$$\lim_{x\to+\infty}f(x)=\frac{\left(5+\lim\limits_{x\to+\infty}\frac{1}{x}\right)\left(\sqrt{1+\lim\limits_{x\to+\infty}\frac{1}{x^2}}+1\right)}{\left(\sqrt[3]{8+\lim\limits_{x\to+\infty}\frac{5}{x^2}+\lim\limits_{x\to+\infty}\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\lim\limits_{x\to+\infty}\frac{5}{x^2}+\lim\limits_{x\to+\infty}\frac{1}{x^3}}+4}=$$

$$\frac{(5+0)\left(\sqrt{1+0}+1\right)}{\left(\sqrt[3]{8+0+0}\right)^2+2\sqrt[3]{8+0+0}+4}=\frac{5\bullet 2}{3\bullet 4}=\frac{5}{6}$$

4.

Δίνεται η συνάρτηση $f(x)=\sqrt[3]{8x^3+3x+1}+\sqrt{25x^2+5x+7}-7x$

Να βρεθεί το όριο $\lim\limits_{x\to+\infty}f(x)$

$$f\left(x\right)=\sqrt[3]{8x^3+3x+1}+\sqrt{25x^2+5x+7}-7x=$$

$$\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}+\sqrt{x^2\left(25+\frac{5}{x}+\frac{7}{x^2}\right)}-7x=$$

$$\sqrt[3]{x^3}\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+\sqrt{x^2}\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-7x\overset{\substack{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x\\ \sqrt{x^2}=|x|}}{=}$$

$$x\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+|x|\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-7x\overset{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x}{=}$$

$$x\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+x\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-7x=x\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-7\right)$$

Έχω:

$$\lim_{x\to+\infty}x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-7\right)=$$

$$\sqrt[3]{8+\lim_{x\to+\infty}\frac{3}{x^2}+\lim_{x\to+\infty}\frac{1}{x^3}}+\sqrt{25+\lim_{x\to+\infty}\frac{5}{x}+\lim_{x\to+\infty}\frac{7}{x^2}}-7=$$

$$\sqrt[3]{8+0+0}+\sqrt{25+0+0}-7=2+5-7=0$$

$$f\left(x\right)=\sqrt[3]{8x^3+3x+1}+\sqrt{25x^2+5x+7}-7x\overset{\Theta\acute{\varepsilon}\tau\omega:-7x=-2x-5x}{=}$$

$$\left(\sqrt[3]{8x^3+3x+1}-2x\right)+\left(\sqrt{25x^2+5x+7}-5x\right)$$

$$\sqrt[3]{8x^3+3x+1}-2x=\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}-2x=\sqrt[3]{x^3}\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}-2x\overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=}$$

$$x\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}-2x=x\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}-2\right)$$

Έχω:

$$\lim_{x\to+\infty}x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}-2\right)=\sqrt[3]{8+\lim_{x\to+\infty}\frac{3}{x^2}+\lim_{x\to+\infty}\frac{1}{x^3}}-2=\sqrt[3]{8+0+0}-2=2-2=0$$

$$\sqrt[3]{8x^3+3x+1}-2x=\frac{\left(\sqrt[3]{8x^3+3x+1}-2x\right)\left[\left(\sqrt[3]{8x^3+3x+1}\right)^2+\sqrt[3]{8x^3+3x+1}\bullet 2x+\left(2x\right)^2\right]}{\left(\sqrt[3]{8x^3+3x+1}\right)^2+\sqrt[3]{8x^3+3x+1}\bullet 2x+\left(2x\right)^2}$$

$$\overset{(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)=\alpha^3-\beta^3}{=} \frac{\left(\sqrt[3]{8x^3+3x+1}\right)^3-(2x)^3}{\left(\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}\right)^2+2x\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}+4x^2}=$$

$$\frac{8x^3+3x+1-8x^3}{\left(\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}\right)^2+2x\sqrt[3]{x^3\left(8+\frac{3}{x^2}+\frac{1}{x^3}\right)}+4x^2}=$$

$$\frac{x\left(3+\frac{1}{x}\right)}{\left(\sqrt[3]{x^3}\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2x\sqrt[3]{x^3}\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4x^2} \overset{x\to+\infty\Rightarrow\sqrt[3]{x^3}=x}{=}$$

$$\frac{x\left(3+\frac{1}{x}\right)}{\left(x\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2x\bullet x\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4x^2}=$$

$$\frac{x\left(3+\frac{1}{x}\right)}{x^2\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2x^2\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4x^2}=$$

$$\frac{x\left(3+\frac{1}{x}\right)}{x^2\left[\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4\right]}=\frac{1}{x}\frac{3+\frac{1}{x}}{\left(\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\frac{3}{x^2}+\frac{1}{x^3}}+4}$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{8x^3+3x+1}-2x\right)=$$

$$\lim_{x\to+\infty}\frac{1}{x}\frac{3+\lim\limits_{x\to+\infty}\frac{1}{x}}{\left(\sqrt[3]{8+\lim\limits_{x\to+\infty}\frac{3}{x^2}+\lim\limits_{x\to+\infty}\frac{1}{x^3}}\right)^2+2\sqrt[3]{8+\lim\limits_{x\to+\infty}\frac{3}{x^2}+\lim\limits_{x\to+\infty}\frac{1}{x^3}}+4}=$$

$$=0\bullet\frac{3+0}{\left(\sqrt[3]{8+0+0}\right)^2+2\sqrt[3]{8+0+0}+4}=0$$

$$\sqrt{25x^2+5x+7}-5x=\sqrt{x^2\left(25+\frac{5}{x}+\frac{7}{x^2}\right)}-5x=\sqrt{x^2}\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-5x \overset{\sqrt{x^2}=|x|}{=}$$

$$|x|\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-5x \overset{x\to+\infty \Rightarrow |x|=x}{=} x\left(\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-5\right)$$

Έχω:

$$\lim_{x\to+\infty} x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}-5\right)=\sqrt{25+\lim_{x\to+\infty}\frac{5}{x}+\lim_{x\to+\infty}\frac{7}{x^2}}-5=\sqrt{25+0+0}-5=5-5=0$$

$$\sqrt{25x^2+5x+7}-5x=\frac{\left(\sqrt{25x^2+5x+7}-5x\right)\left(\sqrt{25x^2+5x+7}+5x\right)}{\sqrt{25x^2+5x+7}+5x}\overset{(\alpha-\beta)(\alpha+\beta)=\alpha^2-\beta^2}{=}$$

$$\frac{\left(\sqrt{25x^2+5x+7}\right)^2-(5x)^2}{\sqrt{x^2\left(25+\frac{5}{x}+\frac{7}{x^2}\right)}+5x}=\frac{25x^2+5x+7-25x^2}{\sqrt{x^2}\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}+5x}\overset{\sqrt{x^2}=|x|}{=}\frac{x\left(5+\frac{7}{x}\right)}{|x|\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}+5x}$$

$$\overset{x\to+\infty\Rightarrow|x|=x}{=}\frac{x\left(5+\frac{7}{x}\right)}{x\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}+5x}=\frac{\not{x}\left(5+\frac{7}{x}\right)}{\not{x}\left(\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}+5\right)}=\frac{5+\frac{7}{x}}{\sqrt{25+\frac{5}{x}+\frac{7}{x^2}}+5}$$

$$\lim_{x\to+\infty}\left(\sqrt{25x^2+5x+7}-5x\right)=\frac{5+\lim_{x\to+\infty}\frac{7}{x}}{\sqrt{25+\lim_{x\to+\infty}\frac{5}{x}+\lim_{x\to+\infty}\frac{7}{x^2}}+5}=\frac{5+0}{\sqrt{25+0+0}+5}=\frac{5}{2\bullet 5}=\frac{1}{2}$$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\left[\left(\sqrt[3]{8x^3+3x+1}-2x\right)+\left(\sqrt{25x^2+5x+7}-5x\right)\right]=$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{8x^3+3x+1}-2x\right)+\lim_{x\to+\infty}\left(\sqrt{25x^2+5x+7}-5x\right)=0+\frac{1}{2}=\frac{1}{2}$$

5.

Δίνεται η συνάρτηση $f(x)=\sqrt[3]{27x^3+1}-\sqrt{9x^2+5}$. *Να βρεθεί το όριο* $\lim_{x\to+\infty} f(x)$

$$f(x)=\sqrt[3]{27x^3+1}-\sqrt{9x^2+5}=\sqrt[3]{x^3\left(27+\frac{1}{x^3}\right)}-\sqrt{x^2\left(9+\frac{5}{x^2}\right)}=$$

$$\sqrt[3]{x^3}\sqrt[3]{27+\frac{1}{x^3}}-\sqrt{x^2}\sqrt{9+\frac{5}{x^2}}\overset{\substack{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x\\ \sqrt{x^2}=|x|}}{=}x\sqrt[3]{27+\frac{1}{x^3}}-|x|\sqrt{9+\frac{5}{x^2}}\overset{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x}{=}$$

$$x\left(\sqrt[3]{27+\frac{1}{x^3}}-\sqrt{9+\frac{5}{x^2}}\right)$$

Έχω:

$$\lim_{x\to+\infty} x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{27+\frac{1}{x^3}}-\sqrt{9+\frac{5}{x^2}}\right)=\sqrt[3]{27+\lim_{x\to+\infty}\frac{1}{x^3}}-\sqrt{9+\lim_{x\to+\infty}\frac{5}{x^2}}=\sqrt[3]{27+0}-\sqrt{9+0}=3-3=0$$

$$f\left(x\right)=\sqrt[3]{27x^3+1}-\sqrt{9x^2+5}=\sqrt[3]{27x^3+1}-3x+3x-\sqrt{9x^2+5}=$$

$$\left(\sqrt[3]{27x^3+1}-3x\right)-\left(\sqrt{9x^2+5}-3x\right)$$

$$\sqrt[3]{27x^3+1}-3x=\sqrt[3]{x^3\left(27+\frac{1}{x^3}\right)}-3x=\sqrt[3]{x^3}\sqrt[3]{27+\frac{1}{x^3}}-3x\overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=}$$

$$x\sqrt[3]{27+\frac{1}{x^3}}-3x=x\left(\sqrt[3]{27+\frac{1}{x^3}}-3\right)$$

Έχω:

$$\lim_{x\to+\infty}x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{27+\frac{1}{x^3}}-3\right)=\sqrt[3]{27+\lim_{x\to+\infty}\frac{1}{x^3}}-3=3-3=0$$

$$\sqrt[3]{27x^3+1}-3x=\frac{\left(\sqrt[3]{27x^3+1}-3x\right)\left[\left(\sqrt[3]{27x^3+1}\right)^2+\sqrt[3]{27x^3+1}\bullet3x+\left(3x\right)^2\right]}{\left(\sqrt[3]{27x^3+1}\right)^2+\sqrt[3]{27x^3+1}\bullet3x+\left(3x\right)^2}$$

$$\overset{(\alpha-\beta)\left(\alpha^2+\alpha\beta+\beta^2\right)=\alpha^3-\beta^3}{=}\frac{\left(\sqrt[3]{27x^3+1}\right)^3-\left(3x\right)^3}{\left(\sqrt[3]{x^3\left(27+\frac{1}{x^3}\right)}\right)^2+3x\sqrt[3]{x^3\left(27+\frac{1}{x^3}\right)}+9x^2}=$$

$$\frac{27x^3+1-27x^3}{\left(\sqrt[3]{x^3}\sqrt[3]{27+\frac{1}{x^3}}\right)^2+3x\sqrt[3]{x^3}\sqrt[3]{27+\frac{1}{x^3}}+9x^2}\overset{x\to+\infty\Rightarrow|x|=x}{=}$$

$$\frac{27x^3+1-27x^3}{\left(x\sqrt[3]{27+\frac{1}{x^3}}\right)^2+3x\bullet x\sqrt[3]{27+\frac{1}{x^3}}+9x^2}=\frac{1}{x^2\left(\sqrt[3]{27+\frac{1}{x^3}}\right)^2+3x^2\sqrt[3]{27+\frac{1}{x^3}}+9x^2}=$$

$$\frac{1}{x^2\left[\left(\sqrt[3]{27+\frac{1}{x^3}}\right)^2+3\sqrt[3]{27+\frac{1}{x^3}}+9\right]}=\frac{1}{x^2}\frac{1}{\left(\sqrt[3]{27+\frac{1}{x^3}}\right)^2+3\sqrt[3]{27+\frac{1}{x^3}}+9}$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{27x^3+1}-3x\right)=\lim_{x\to+\infty}\frac{1}{x^2}\frac{1}{\left(\sqrt[3]{27+\lim_{x\to+\infty}\frac{1}{x^3}}\right)^2+3\sqrt[3]{27+\lim_{x\to+\infty}\frac{1}{x^3}}+9}=$$

$$=0\bullet\frac{1}{\left(\sqrt[3]{27+0}\right)^2+3\sqrt[3]{27+0}+9}=0$$

$$\sqrt{9x^2+5}-3x=\sqrt{x^2\left(9+\frac{5}{x^2}\right)}-3x=\sqrt{x^2}\sqrt{9+\frac{5}{x^2}}-3x\overset{\sqrt{x^2}=|x|}{=}|x|\sqrt{9+\frac{5}{x^2}}-3x\overset{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x}{=}$$

$$x\sqrt{9+\frac{5}{x^2}}-3x=x\left(\sqrt{9+\frac{5}{x^2}}-3\right)$$

$Έχω:$

$$\lim_{x\to+\infty}x=+\infty,\ \lim_{x\to+\infty}\left(\sqrt{9+\frac{5}{x^2}}-3\right)=\sqrt{9+\lim_{x\to+\infty}\frac{5}{x^2}}-3=3-3=0$$

$$\sqrt{9x^2+5}-3x=\frac{\left(\sqrt{9x^2+5}-3x\right)\left(\sqrt{9x^2+5}+3x\right)}{\sqrt{9x^2+5}+3x}=\frac{\left(\sqrt{9x^2+5}\right)^2-(3x)^2}{\sqrt{x^2\left(9+\frac{5}{x^2}\right)}+x}=$$

$$=\frac{9x^2+5-9x^2}{\sqrt{x^2}\sqrt{9+\frac{5}{x^2}}+x}\overset{\sqrt{x^2}=|x|}{=}\frac{5}{|x|\sqrt{9+\frac{5}{x^2}}+x}\overset{x\to+\infty\Rightarrow|x|=x}{=}\frac{5}{x\sqrt{9+\frac{5}{x^2}}+x}=\frac{1}{x}\frac{5}{\sqrt{9+\frac{5}{x^2}}+1}$$

$$\lim_{x\to+\infty}\left(\sqrt{9x^2+5}-3x\right)=\lim_{x\to+\infty}\frac{1}{x}\frac{5}{\sqrt{9+\lim_{x\to+\infty}\frac{5}{x^2}}+1}=0\bullet\frac{5}{\sqrt{9+0}+1}=0$$

$$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}\left(\sqrt[3]{27x^3+1}-3x\right)-\lim_{x\to+\infty}\left(\sqrt{9x^2+5}-3x\right)=0-0=0$$

ΑΣΚΗΣΕΙΣ

1.

$Δίνεται\ η\ συνάρτηση\ f(x)=\sqrt[3]{27x^3+5x^2+3}-3x.$ Να βρεθεί το όριο $\lim_{x\to+\infty}f(x)$

2.

$Δίνεται\ η\ συνάρτηση\ f(x)=\frac{\sqrt[3]{64x^3+x+2}-4x}{x+1}.$ Να βρεθεί το όριο $\lim_{x\to+\infty}f(x)$

3.

$Δίνεται\ η\ συνάρτηση\ f(x)=\frac{\sqrt[3]{27x^3+3x^2+5}-3x}{\sqrt{4x^2+1}-2x}.$ Να βρεθεί το όριο $\lim_{x\to+\infty}f(x)$

4.

$Δίνεται\ η\ συνάρτηση\ f(x)=\sqrt[3]{64x^3+1}+\sqrt{25x^2+3}-9x.$ Να βρεθεί το όριο $\lim_{x\to+\infty}f(x)$

5.

$Δίνεται\ η\ συνάρτηση\ f(x)=\sqrt[3]{64x^3+1}-\sqrt{16x^2+3}.$ Να βρεθεί το όριο $\lim_{x\to+\infty}f(x)$