ΕΥΡΕΣΗ ΟΡΙΟΥ ΣΥΝΑΡΤΗΣΗΣ ΜΕ ΠΑΡΑΜΕΤΡΟ ΟΤΑΝ $x \to \pm\infty$

ΠΑΡΑΔΕΙΓΜΑΤΑ

1.

$\Delta\acute{\iota}\nu\varepsilon\tau\alpha\iota\ \eta\ \sigma\upsilon\nu\acute{\alpha}\rho\tau\eta\sigma\eta\ f(x)=\dfrac{(\lambda^2-\lambda-2)x^3-(\lambda-1)x^2+2x+3}{(\lambda-1)x^2+\lambda x+1}$

Να βρεθεί το $\lim\limits_{x\to-\infty} f(x)$

$\Delta\iota\alpha\kappa\rho\acute{\iota}\nu\omega\ \tau\iota\varsigma\ \pi\varepsilon\rho\iota\pi\tau\acute{\omega}\sigma\varepsilon\iota\varsigma:\left\{\begin{matrix}(\mathrm{I})\,\lambda-1=0\\(\mathrm{II})\,\lambda^2-\lambda-2=0\\(\mathrm{III})\,\lambda^2-\lambda-2\neq 0,\lambda-1\neq 0\end{matrix}\right\}$

<u>Περίπτωση (I)</u>

$\lambda-1=0\Leftrightarrow\lambda=1$

Αν $\lambda=1$ ο τύπος της f γίνεται:

$$f(x)=\frac{(\lambda^2-\lambda-2)x^3-(\lambda-1)x^2+2x+3}{(\lambda-1)x^2+\lambda x+1}\overset{\lambda=1}{=}\frac{(1^2-1-2)x^3-(1-1)x^2+2x+3}{(1-1)x^2+1x+1}=$$

$$=\frac{-2x^3+2x+3}{x+1}$$

$$\lim_{x\to-\infty}f(x)=\lim_{x\to-\infty}\frac{-2x^3+2x+3}{x+1}=\lim_{x\to-\infty}\frac{-2x^3}{x}=-2\lim_{x\to-\infty}x^2=-2(+\infty)=-\infty$$

<u>Περίπτωση (I)</u>

$\lambda^2-\lambda-2=0\,(1)$

$\Delta=\beta^2-4\alpha\gamma=(-1)^2-4\cdot1\cdot(-2)=1+8=9>0$

Επειδή $\Delta>0$ η δευτεροβάθμια εξίσωση (1) έχει δυο ρίζες πραγματικές και άνισες:

$$\lambda_{1,2}=\frac{-\beta\pm\sqrt{\Delta}}{2\alpha}=\frac{1\pm\sqrt{9}}{2}=\frac{1\pm3}{2}=\begin{cases}\nearrow\frac{1+3}{2}=\frac{4}{2}=2\\ \searrow\frac{1-3}{2}=\frac{-2}{2}=-1\end{cases}$$

Αν $\lambda=2$ ο τύπος της f γίνεται:

$$f(x)=\frac{(\lambda^2-\lambda-2)x^3-(\lambda-1)x^2+2x+3}{(\lambda-1)x^2+\lambda x+1}\overset{\lambda=2}{=}\frac{(2^2-2-2)x^3-(2-1)x^2+2x+3}{(2-1)x^2+2x+1}=$$

$$=\frac{-x^2+2x+3}{x^2+2x+1}$$

$$\lim_{x\to-\infty}f(x)=\lim_{x\to-\infty}\frac{-x^2+2x+3}{x^2+2x+1}=\lim_{x\to-\infty}\frac{-x^2}{x^2}=-1$$

$A\nu\ \lambda=-1\ o\ \tau\acute{\upsilon}\pi o\varsigma\ \tau\eta\varsigma\ f\ \gamma\acute{\iota}\nu\varepsilon\tau\alpha\iota:$

$$f(x)=\frac{(\lambda^2-\lambda-2)x^3-(\lambda-1)x^2+2x+3}{(\lambda-1)x^2+\lambda x+1}\overset{\lambda=-1}{=}\frac{\left[(-1)^2-(-1)-2\right]x^3-(-1-1)x^2+2x+3}{(-1-1)x^2+2x+1}$$

$$=\frac{2x^2+2x+3}{-2x^2+2x+1}$$

$$\lim_{x\to-\infty}f(x)=\lim_{x\to-\infty}\frac{2x^2+2x+3}{-2x^2+2x+1}=\lim_{x\to-\infty}\frac{2x^2}{-2x^2}=-1$$

$\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta\ (\mathrm{III})$

$$\left\{\begin{matrix}\lambda^2-\lambda-2\neq 0\\ \lambda-1\neq 0\end{matrix}\right\}\Leftrightarrow\lambda\neq\pm1,2$$

$$\lim_{x\to-\infty}f(x)=\lim_{x\to-\infty}\frac{(\lambda^2-\lambda-2)x^3-(\lambda-1)x^2+2x+3}{(\lambda-1)x^2+\lambda x+1}=\lim_{x\to-\infty}\frac{(\lambda^2-\lambda-2)x^3}{(\lambda-1)x^2}=$$

$$=\frac{\lambda^2-\lambda-2}{\lambda-1}\lim_{x\to-\infty}x=\frac{\lambda^2-\lambda-2}{\lambda-1}(-\infty)=\begin{cases}-\infty,\ \dfrac{\lambda^2-\lambda-2}{\lambda-1}>0,\lambda\neq\pm1,2\\ +\infty,\ \dfrac{\lambda^2-\lambda-2}{\lambda-1}<0,\lambda\neq\pm1,2\end{cases}$$

λ	$-\infty$	-1		1		2	$+\infty$
$\lambda^2-\lambda-2$	$+$	‖	$-$	‖	$-$	‖	$+$
$\lambda-1$	$-$		$-$		$+$		$+$
$\dfrac{\lambda^2-\lambda-2}{\lambda-1}$	$-$	‖	$+$	‖	$-$	‖	$+$

$$=\begin{cases}-\infty,\ \lambda\in(-1,1)\cup(2,+\infty)\\ +\infty,\ \lambda\in(-\infty,-1)\cup(1,2)\end{cases}$$

2.

> $\Delta\acute{\iota}\nu\varepsilon\tau\alpha\iota\ \eta\ \sigma\upsilon\nu\acute{\alpha}\rho\tau\eta\sigma\eta\ f(x)=\sqrt[3]{-x^3+3x+2}+\lambda x$
>
> $N\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \lim\limits_{x\to-\infty}f(x)$

$$x\to-\infty\Rightarrow x<0\Rightarrow -x>0\Rightarrow\sqrt[3]{(-x)^3}=-x$$

$$f(x)=\sqrt[3]{-x^3+3x+2}+\lambda x=\sqrt[3]{-x^3\left(1+\frac{3x}{-x^3}+\frac{2}{-x^3}\right)}+\lambda x\overset{(-a)^3=-a^3}{=}$$

$$\sqrt[3]{(-x)^3\left(1-\frac{3}{x^2}-\frac{2}{x^3}\right)}+\lambda x=\sqrt[3]{(-x)^3}\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+\lambda x=-x\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+\lambda x=$$

$$=x\left(-\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+\lambda\right)$$

Έχω:

$$\lim_{x\to-\infty}x=-\infty$$

$$\lim_{x\to-\infty}\left(-\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+\lambda\right)=\lim_{x\to-\infty}\left(-\sqrt[3]{1-\lim_{x\to-\infty}\frac{3}{x^2}-\lim_{x\to-\infty}\frac{2}{x^3}}+\lambda\right)=-1+\lambda$$

Διακρίνω τις περιπτώσεις: $\begin{Bmatrix}(\mathrm{I})\,-1+\lambda=0\\(\mathrm{II})\,-1+\lambda\neq 0\end{Bmatrix}$

<u>*Περίπτωση* (I)</u>

$-1+\lambda=0\Leftrightarrow\lambda=1$

Αν $\lambda=1$ *ο τύπος της* f *γίνεται*:

$$f(x)=\sqrt[3]{-x^3+3x+2}+\lambda x\overset{\lambda=1}{=}\sqrt[3]{-x^3+3x+2}+x=$$

$$\frac{\left(\sqrt[3]{-x^3+3x+2}+x\right)\left[\left(\sqrt[3]{-x^3+3x+2}\right)^2-x\sqrt[3]{-x^3+3x+2}+x^2\right]}{\left(\sqrt[3]{-x^3+3x+2}\right)^2-x\sqrt[3]{-x^3+3x+2}+x^2}\overset{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=\alpha^3+\beta^3}{=}$$

$$\frac{\left(\sqrt[3]{-x^3+3x+2}\right)^3+x^3}{\left(\sqrt[3]{-x^3\left(1+\frac{3x}{-x^3}+\frac{2}{-x^3}\right)}\right)^2-x\sqrt[3]{-x^3\left(1+\frac{3x}{-x^3}+\frac{2}{-x^3}\right)}+x^2}=$$

$$\frac{-x^3+3x+2+x^3}{\left(\sqrt[3]{-x^3\left(1+\frac{3x}{-x^3}+\frac{2}{-x^3}\right)}\right)^2-x\sqrt[3]{-x^3\left(1+\frac{3x}{-x^3}+\frac{2}{-x^3}\right)}+x^2}\overset{(-a)^3=-a^3}{=}$$

$$\frac{x\left(3+\frac{2}{x}\right)}{\left(\sqrt[3]{(-x)^3}\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}\right)^2-x\sqrt[3]{(-x)^3}\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+x^2}\overset{x\to-\infty\Rightarrow x<0\Rightarrow -x>0\Rightarrow\sqrt[3]{(-x)^3}=-x}{=}$$

$$\frac{x\left(3+\frac{2}{x}\right)}{\left(-x\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}\right)^2-x(-x)\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+x^2}=$$

$$\frac{x\left(3+\frac{2}{x}\right)}{x^2\left(\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}\right)^2+x^2\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+x^2}=$$

$$\frac{x\left(3+\frac{2}{x}\right)}{x^2\left[\left(\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}\right)^2+\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+1\right]}=$$

$$=\frac{1}{x}\frac{3+\frac{2}{x}}{\left(\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}\right)^2+\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+1}$$

$$\lim_{x\to-\infty}f(x)=\lim_{x\to-\infty}\frac{1}{x}\frac{3+\lim\limits_{x\to-\infty}\frac{2}{x}}{\left(\sqrt[3]{1-\lim\limits_{x\to-\infty}\frac{3}{x^2}-\lim\limits_{x\to-\infty}\frac{2}{x^3}}\right)^2+\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+1}=$$

$$=0\bullet\frac{3+0}{\left(\sqrt[3]{1-0-0}\right)^2+\sqrt[3]{1-0-0}+1}=0$$

$\underline{\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta\,(\mathrm{II})}$

$-1+\lambda\neq0\Leftrightarrow\lambda\neq1$

$$\lim_{x\to-\infty}f(x)=\lim_{x\to-\infty}x\left(-\sqrt[3]{1-\frac{3}{x^2}-\frac{2}{x^3}}+\lambda\right)=\lim_{x\to-\infty}x\left(-\sqrt[3]{1-\lim\limits_{x\to-\infty}\frac{3}{x^2}-\lim\limits_{x\to-\infty}\frac{2}{x^3}}+\lambda\right)=$$

$$(-\infty)(\lambda-1)=\begin{cases}-\infty,\lambda-1>0\\+\infty,\lambda-1>0\end{cases}=\begin{cases}-\infty,\lambda>1\\+\infty,\lambda<1\end{cases}$$

3.

Δίνεται η συνάρτηση $f(x)=\sqrt{x^2-5x+6}-\lambda x+3$
Να βρεθεί το $\lim\limits_{x\to+\infty}f(x)$

$x\to+\infty\Rightarrow x>0\Rightarrow|x|=x$

$$f(x)=\sqrt{x^2-5x+6}-\lambda x+3=\sqrt{x^2\left(1-\frac{5}{x}+\frac{6}{x^2}\right)}-x\left(\lambda-\frac{3}{x}\right)=$$

$$\sqrt{x^2}\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}-x\left(\lambda-\frac{3}{x}\right)\overset{\sqrt{x^2}=|x|}{=}|x|\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}-x\left(\lambda-\frac{3}{x}\right)\overset{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x}{=}$$

$$x\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}-x\left(\lambda-\frac{3}{x}\right)=x\left(\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}-\lambda+\frac{3}{x}\right)$$

Έχω:

$$\lim_{x\to+\infty}x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}-\lambda+\frac{3}{x}\right)=\lim_{x\to+\infty}\left(\sqrt{1-\lim_{x\to+\infty}\frac{5}{x}+\lim_{x\to+\infty}\frac{6}{x^2}}-\lambda+\lim_{x\to+\infty}\frac{3}{x}x\right)=1-\lambda$$

Διακρίνω τις περιπτώσεις: $\left\{\begin{matrix}(\text{I})1-\lambda=0\\(\text{II})1-\lambda\neq0\end{matrix}\right\}$

<u>*Περίπτωση* (I):</u>

$1-\lambda=0\Leftrightarrow\lambda=1$

Αν $\lambda=1$ *ο τύπος της* f *γίνεται*:

$$f(x)=\sqrt{x^2-5x+6}-\lambda x+3\overset{\lambda=1}{=}\sqrt{x^2-5x+6}-x+3=$$

$$\frac{\left[\sqrt{x^2-5x+6}-(x-3)\right]\left[\sqrt{x^2-5x+6}+(x-3)\right]}{\sqrt{x^2-5x+6}+(x-3)}=$$

$$\frac{\left(\sqrt{x^2-5x+6}\right)^2-(x-3)^2}{\sqrt{x^2\left(1-\frac{5}{x}+\frac{6}{x^2}\right)}+x\left(1-\frac{3}{x}\right)}=\frac{\sqrt{x^2-5x+6}-\left(x^2-6x+9\right)}{\sqrt{x^2}\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}+x\left(1-\frac{3}{x}\right)}\overset{\sqrt{x^2}=|x|}{=}$$

$$\frac{x-3}{|x|\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}+x\left(1-\frac{3}{x}\right)}\overset{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x}{=}\frac{x\left(1-\frac{3}{x}\right)}{x\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}+x\left(1-\frac{3}{x}\right)}=$$

$$\frac{\cancel{x}\left(1-\frac{3}{x}\right)}{\cancel{x}\left(\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}+1-\frac{3}{x}\right)}=\frac{1-\frac{3}{x}}{\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}+1-\frac{3}{x}}$$

$$\lim_{x\to+\infty}f(x)=\frac{1-\lim_{x\to+\infty}\frac{3}{x}}{\sqrt{1-\lim_{x\to+\infty}\frac{5}{x}+\lim_{x\to+\infty}\frac{6}{x^2}}+1-\lim_{x\to+\infty}\frac{3}{x}}=\frac{1-0}{\sqrt{1-0+0}+1-0}=\frac{1}{2}$$

$\underline{\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta\,(\mathrm{II}):}$

$1-\lambda \neq 0 \Leftrightarrow \lambda \neq 1$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x\left(\sqrt{1-\lim_{x\to+\infty}\frac{5}{x}+\lim_{x\to+\infty}\frac{6}{x^2}}-\lambda+\lim_{x\to+\infty}\frac{3}{x}\right) = (+\infty)(1-\lambda) =$$

$$\begin{cases} +\infty, 1-\lambda > 0 \\ -\infty, 1-\lambda < 0 \end{cases} = \begin{cases} +\infty, \lambda < 1 \\ -\infty, \lambda > 1 \end{cases}$$

4.

$\Delta\acute{\iota}\nu\varepsilon\tau\alpha\iota\ \eta\ \sigma\upsilon\nu\acute{\alpha}\rho\tau\eta\sigma\eta\ f(x) = \sqrt[3]{x^3+1}+\sqrt{x^2+x}-\lambda x$
$\mathrm{N}\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o \lim_{x\to+\infty} f(x)$

$$f(x) = \sqrt[3]{x^3+1}+\sqrt{x^2+x}-\lambda x = \sqrt[3]{x^3\left(1+\frac{1}{x^3}\right)}+\sqrt{x^2\left(1+\frac{6}{x^2}\right)}-\lambda x =$$

$$\sqrt[3]{x^3}\sqrt[3]{1+\frac{1}{x^3}}+\sqrt{x^2}\sqrt{1+\frac{6}{x^2}}-\lambda x \overset{\substack{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x \\ \sqrt{x^2}=|x|}}{=} x\sqrt[3]{1+\frac{1}{x^3}}+|x|\sqrt{1+\frac{6}{x^2}}-\lambda x$$

$$\overset{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x}{=} x\sqrt[3]{1+\frac{1}{x^3}}+x\sqrt{1+\frac{6}{x^2}}-\lambda x = x\left(\sqrt[3]{1+\frac{1}{x^3}}+\sqrt{1+\frac{6}{x^2}}-\lambda\right)$$

$\acute{E}\chi\omega:$

$$\lim_{x\to+\infty} x = +\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{1+\frac{1}{x^3}}+\sqrt{1+\frac{6}{x^2}}-\lambda\right) = \sqrt[3]{1+\lim_{x\to+\infty}\frac{1}{x^3}}+\sqrt{1+\lim_{x\to+\infty}\frac{6}{x^2}}-\lambda = 2-\lambda$$

$$\Delta\iota\alpha\kappa\rho\acute{\iota}\nu\omega\ \tau\iota\varsigma\ \pi\varepsilon\rho\iota\pi\tau\acute{\omega}\sigma\varepsilon\iota\varsigma: \begin{Bmatrix} (\mathrm{I})\, 2-\lambda = 0 \\ (\mathrm{II})\, 2-\lambda \neq 0 \end{Bmatrix}$$

$\underline{\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta\,(\mathrm{I}):}$

$2-\lambda = 0 \Leftrightarrow \lambda = 2$

$\mathrm{A}\nu\ \lambda = 2\ o\ \tau\acute{\upsilon}\pi o\varsigma\ \tau\eta\varsigma\ f\ \gamma\acute{\iota}\nu\varepsilon\tau\alpha\iota:$

$$f(x) = \sqrt[3]{x^3+1}+\sqrt{x^2+x}-\lambda x \overset{\lambda=2}{=} \sqrt[3]{x^3+1}+\sqrt{x^2+x}-2x = \left(\sqrt[3]{x^3+1}-x\right)+\left(\sqrt{x^2+x}-x\right)$$

$$\sqrt[3]{x^3+1}-x = \sqrt[3]{x^3\left(1+\frac{1}{x^3}\right)}-x = \sqrt[3]{x^3}\sqrt[3]{1+\frac{1}{x^3}}-x \overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=}$$

$$x\sqrt[3]{1+\frac{1}{x^3}}-x = x\left(\sqrt[3]{1+\frac{1}{x^3}}-1\right)$$

$Έχω:$

$$\lim_{x\to+\infty} x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{1+\frac{1}{x^3}}-1\right)=\sqrt[3]{1+\lim_{x\to+\infty}\frac{1}{x^3}}-1=0$$

$$\sqrt[3]{x^3+1}-x=\frac{\left(\sqrt[3]{x^3+1}-x\right)\left[\left(\sqrt[3]{x^3+1}\right)^2+x\sqrt[3]{x^3+1}+x^2\right]}{\left(\sqrt[3]{x^3+1}\right)^2+x\sqrt[3]{x^3+1}+x^2}\overset{(\alpha-\beta)\left(\alpha^2+\alpha\beta+\beta^2\right)=\alpha^3-\beta^3}{=}$$

$$\frac{\left(\sqrt[3]{x^3+1}\right)^3-x^3}{\left(\sqrt[3]{x^3\left(1+\frac{1}{x^3}\right)}\right)^2+x\sqrt[3]{x^3\left(1+\frac{1}{x^3}\right)}+x^2}=\frac{1}{\left(\sqrt[3]{x^3}\sqrt[3]{1+\frac{1}{x^3}}\right)^2+x\sqrt[3]{x^3}\sqrt[3]{1+\frac{1}{x^3}}+x^2}$$

$$\overset{x\to+\infty\Rightarrow x>0\Rightarrow\sqrt[3]{x^3}=x}{=}\frac{1}{x^2\left(\sqrt[3]{1+\frac{1}{x^3}}\right)^2+x^2\sqrt[3]{1+\frac{1}{x^3}}+x^2}=\frac{1}{x^2}\frac{1}{\left(\sqrt[3]{1+\frac{1}{x^3}}\right)^2+\sqrt[3]{1+\frac{1}{x^3}}+1}$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{x^3+1}-x\right)=\lim_{x\to+\infty}\frac{1}{x^2}\frac{1}{\left(\sqrt[3]{1+\lim\limits_{x\to+\infty}\frac{1}{x^3}}\right)^2+\sqrt[3]{1+\lim\limits_{x\to+\infty}\frac{1}{x^3}}+1}=0\frac{1}{\left(\sqrt[3]{1+0}\right)^2+\sqrt[3]{1+0}+1}=0$$

$$\sqrt{x^2+x}-x=\sqrt{x^2\left(1+\frac{1}{x}\right)}-x=\sqrt{x^2}\sqrt{1+\frac{1}{x}}-x\overset{\sqrt{x^2}=|x|}{=}|x|\sqrt{1+\frac{1}{x}}-x\overset{x\to+\infty\Rightarrow x>0\Rightarrow|x|=x}{=}$$

$$x\sqrt{1+\frac{1}{x}}-x=x\left(\sqrt{1+\frac{1}{x}}-1\right)$$

$Έχω:$

$$\lim_{x\to+\infty} x=+\infty$$

$$\lim_{x\to+\infty}\left(\sqrt{1+\frac{1}{x}}-1\right)=\sqrt{1+\lim_{x\to+\infty}\frac{1}{x}}-1=\sqrt{1+0}-1=1-1=0$$

$$\sqrt{x^2+x}-x=\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+x\right)}{\sqrt{x^2+x}+x}=\frac{\left(\sqrt{x^2+x}\right)^2-x^2}{\sqrt{x^2+x}+x}=$$

$$\frac{1}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}+x}=\frac{1}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}+x}\overset{\sqrt{x^2}=|x|}{=}\frac{1}{|x|\sqrt{1+\frac{1}{x^2}}+x}\overset{x\to+\infty\Rightarrow|x|=x}{=}$$

$$\frac{1}{x\left(\sqrt{1+\frac{1}{x^2}}+1\right)}=\frac{1}{x}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1}$$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+x}-x\right)=\lim_{x\to+\infty}\frac{1}{x}\frac{1}{\sqrt{1+\lim_{x\to+\infty}\frac{1}{x^2}}+1}=0\frac{1}{\sqrt{1+0}+1}=0$$

$$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}\left(\sqrt[3]{x^3+1}-x\right)+\lim_{x\to+\infty}\left(\sqrt{x^2+x}-x\right)=0$$

$\underline{\Pi\varepsilon\rho\acute{\iota}\pi\tau\omega\sigma\eta\,(\mathrm{II}):}$

$2-\lambda\neq 0\Leftrightarrow\lambda\neq 2$

$$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}x\left(\sqrt[3]{1+\lim_{x\to+\infty}\frac{1}{x^3}}+\sqrt{1+\lim_{x\to+\infty}\frac{6}{x^2}}-\lambda\right)=(+\infty)(2-\lambda)=$$

$$\begin{cases}+\infty, 2-\lambda>0\\-\infty, 2-\lambda<0\end{cases}=\begin{cases}+\infty, \lambda<2\\-\infty, \lambda>2\end{cases}$$

<u>ΑΣΚΗΣΕΙΣ</u>

1.

Δίνεται η συνάρτηση $f(x)=\dfrac{(\lambda^2-3\lambda+2)x^3-(\lambda+1)x^2+5x+6}{(\lambda+1)x^2+\lambda^2x+1}$

Να βρεθεί το $\lim_{x\to-\infty}f(x)$

2.

Δίνεται η συνάρτηση $f(x)=\sqrt[3]{-x^3+5x+4}+\lambda x+1$

Να βρεθεί το $\lim_{x\to-\infty}f(x)$

3.

Δίνεται η συνάρτηση $f(x)=\sqrt{x^2-7x+10}-\lambda x+5$

Να βρεθεί το $\lim_{x\to+\infty}f(x)$

4.

Δίνεται η συνάρτηση $f(x)=\sqrt[3]{8x^3+3x+1}+\sqrt{4x^2+4x+3}-\lambda x$

Να βρεθεί το $\lim_{x\to+\infty}f(x)$

5.

Δίνεται η συνάρτηση $f(x)=(\lambda^2-2\lambda)x^3+\lambda x^2+x+5$

Να βρεθεί το $\lim_{x\to+\infty}f(x)$

Υπόδειξη: Διακρίνω τις περιπτώσεις: $(\mathrm{I})\lambda^2-2\lambda=0, (\mathrm{II})\lambda^2-2\lambda\neq 0$

5.

Δίνεται η συνάρτηση $f(x)=\sqrt[3]{x^3+x}-\sqrt{\lambda x^2+1}, \lambda>0$

Να βρεθεί το $\lim_{x\to+\infty}f(x)$

Υπόδειξη: Διακρίνω τις περιπτώσεις: $(\mathrm{I})\lambda=1(\mathrm{II})0<\lambda\neq 1$