ΟΡΙΑ ΜΕ ΧΡΗΣΗ ΤΟΥ ΤΥΠΟΥ $x^{\nu}-1=(x-1)\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)$

ΠΑΡΑΔΕΙΓΜΑΤΑ

1.

$$\text{Να βρεθεί το όριο} \lim_{x\to 0}\frac{\left(x^2+3x+1\right)^7-1}{\left(x^2+2x+1\right)^4-1}$$

$$\text{Έχω}: \lim_{x\to 0}\left[\left(x^2+3x+1\right)^7-1\right]=\lim_{x\to 0}\left[\left(x^2+2x+1\right)^4-1\right]=0$$

$$\frac{\left(x^2+3x+1\right)^7-1}{\left(x^2+2x+1\right)^4-1}=\frac{\left(x^2+3x+1\right)^7-1}{\left(\underbrace{x^2+2\cdot x\cdot 1+1^2}_{\alpha^2+2\cdot\alpha\cdot\beta+\beta^2=(\alpha+\beta)^2}\right)^4-1}=\frac{\left(x^2+3x+1\right)^7-1}{\left[\left(x+1\right)^2\right]^4-1}\overset{\left(\alpha^{\mu}\right)^{\nu}=\alpha^{\mu\nu}}{=}\frac{\left(x^2+3x+1\right)^7-1}{\left(x+1\right)^8-1}$$

$$\overset{x^{\nu}-1=(x-1)\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)}{=}\frac{\left[\left(x^2+3x+1\right)-1\right]\left[\left(x^2+3x+1\right)^6+\left(x^2+3x+1\right)^5+\cdots+\left(x^2+3x+1\right)+1\right]}{\left[\left(x+1\right)-1\right]\left[\left(x+1\right)^7+\left(x+1\right)^6+\cdots+\left(x+1\right)+1\right]}$$

$$=\frac{\cancel{x}\,(x+3)\left[\left(x^2+3x+1\right)^6+\left(x^2+3x+1\right)^5+\cdots+\left(x^2+3x+1\right)+1\right]}{\cancel{x}\left[\left(x+1\right)^7+\left(x+1\right)^6+\cdots+\left(x+1\right)+1\right]}=$$

$$=\frac{(x+3)\left[\left(x^2+3x+1\right)^6+\left(x^2+3x+1\right)^5+\cdots+\left(x^2+3x+1\right)+1\right]}{\left(x+1\right)^7+\left(x+1\right)^6+\cdots+\left(x+1\right)+1}$$

$$\lim_{x\to 0}\frac{\left(x^2+3x+1\right)^7-1}{\left(x^2+3x+1\right)^4-1}=\lim_{x\to 0}\frac{(x+3)\left[\left(x^2+3x+1\right)^6+\left(x^2+3x+1\right)^5+\cdots+\left(x^2+3x+1\right)+1\right]}{\left(x+1\right)^7+\left(x+1\right)^6+\cdots+\left(x+1\right)+1}$$

$$=\frac{3\left(\underbrace{1+1+\cdots+1}_{7-\varphi o\rho\acute{\varepsilon}\varsigma}\right)}{\underbrace{1+1+\cdots+1}_{8-\varphi o\rho\acute{\varepsilon}\varsigma}}=\frac{3\cdot 7}{8}=\frac{21}{8}$$

2.

$$\text{Να βρεθεί το όριο} \lim_{x\to 2}\frac{\left(x^3-7\right)^7-1}{\sqrt[5]{x^2-3}-1}$$

$$\text{Έχω}: \lim_{x\to 2}\left[\left(x^3-7\right)^7-1\right]=\lim_{x\to 2}\left(\sqrt[5]{x^2-3}-1\right)=0$$

$$\frac{\left(x^3-7\right)^7-1}{\sqrt[5]{x^2-3}-1} \overset{x^\nu-1=(x-1)\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)}{=}$$

Πολλαπλασιάζω αριθμητή και παρονομαστή με την παράσταση: $\left(\sqrt[5]{x^2-3}\right)^4+\left(\sqrt[5]{x^2-3}\right)^3+\left(\sqrt[5]{x^2-3}\right)^2+\sqrt[5]{x^2-3}+1$

$$\frac{\left[\left(x^3-7\right)-1\right]\left[\left(x^3-7\right)^6+\cdots+\left(x^3-7\right)+1\right]\left[\left(\sqrt[5]{x^2-3}\right)^4+\cdots+\left(\sqrt[5]{x^2-3}\right)+1\right]}{\left(\sqrt[5]{x^2-3}-1\right)\left[\left(\sqrt[5]{x^2-3}\right)^4+\left(\sqrt[5]{x^2-3}\right)^3+\left(\sqrt[5]{x^2-3}\right)^2+\left(\sqrt[5]{x^2-3}\right)+1\right]}=$$

$$=\frac{\left(x^3-8\right)\left[\left(x^3-7\right)^6+\cdots+\left(x^3-7\right)+1\right]\left[\left(\sqrt[5]{x^2-3}\right)^4+\cdots+\left(\sqrt[5]{x^2-3}\right)+1\right]}{\left[\left(\sqrt[5]{x^2-3}\right)^5-1\right]}=$$

$$\frac{\left(x^3-2^3\right)\left[\left(x^3-7\right)^6+\cdots+\left(x^3-7\right)+1\right]\left[\left(\sqrt[5]{x^2-3}\right)^4+\cdots+\left(\sqrt[5]{x^2-3}\right)+1\right]}{x^2-2^2} \overset{\substack{\alpha^3-\beta^3=(\alpha-\beta)\left(\alpha^2+\alpha\beta+\beta^2\right)\\ \alpha^2-\beta^2=(\alpha-\beta)(\alpha+\beta)}}{=}$$

$$=\frac{\cancel{(x-2)}\left(x^2+2x+2^2\right)\left[\left(x^3-7\right)^6+\cdots+\left(x^3-7\right)+1\right]\left[\left(\sqrt[5]{x^2-3}\right)^4+\cdots+\left(\sqrt[5]{x^2-3}\right)+1\right]}{\cancel{(x-2)}\left(x+2\right)}=$$

$$=\frac{\left(x^2+2x+4\right)\left[\left(x^3-7\right)^6+\cdots+\left(x^3-7\right)+1\right]\left[\left(\sqrt[5]{x^2-3}\right)^4+\cdots+\left(\sqrt[5]{x^2-3}\right)+1\right]}{x+2}$$

$$\lim_{x\to2}\frac{\left(x^3-7\right)^7-1}{\sqrt[5]{x^2-3}-1}=$$

$$\lim_{x\to2}\frac{\left(x^2+2x+4\right)\left[\left(x^3-7\right)^6+\cdots+\left(x^3-7\right)+1\right]\left[\left(\sqrt[5]{x^2-3}\right)^4+\cdots+\left(\sqrt[5]{x^2-3}\right)+1\right]}{x+2}=$$

$$\frac{\left(4+4+4\right)\left(\underbrace{1+1+\cdots+1}_{7-\varphi o\rho\acute{\varepsilon}\varsigma}\right)\left(\underbrace{1+1+\cdots+1}_{5-\varphi o\rho\acute{\varepsilon}\varsigma}\right)}{2+2}=\frac{3\cdot4\cdot7\cdot5}{2\cdot2}=21\cdot5=105$$

3.

Να βρεθεί το όριο $\lim\limits_{x\to3}\dfrac{\sqrt[5]{x^3-26}-1}{\sqrt[4]{2x^2-17}-1}$

$$\text{Έχω}:\lim_{x\to3}\left(\sqrt[5]{x^3-26}-1\right)=\lim_{x\to3}\left(\sqrt[4]{2x^2-17}-1\right)=0$$

Πολλαπλασιάζω αριθμητή και παρονομαστή με την παράσταση:
$\left[\left(\sqrt[5]{x^3-26}\right)^4+\left(\sqrt[5]{x^3-26}\right)^3+\left(\sqrt[5]{x^3-26}\right)^2+\left(\sqrt[5]{x^3-26}\right)+1\right]\left[\left(\sqrt[4]{2x^2-17}\right)^3+\left(\sqrt[4]{2x^2-17}\right)^2+\left(\sqrt[4]{2x^2-17}\right)+1\right]$

$$\frac{\sqrt[5]{x^3-26}-1}{\sqrt[4]{2x^2-17}-1} =$$

$$\frac{\left(\sqrt[5]{x^3-26}-1\right)\left[\left(\sqrt[5]{x^3-26}\right)^4+\cdots+\left(\sqrt[5]{x^3-26}\right)+1\right]\left[\left(\sqrt[4]{2x^2-17}\right)^3+\cdots+\left(\sqrt[4]{2x^2-17}\right)+1\right]}{\left(\sqrt[4]{2x^2-17}-1\right)\left[\left(\sqrt[4]{2x^2-17}\right)^3+\cdots+\left(\sqrt[4]{2x^2-17}\right)+1\right]\left[\left(\sqrt[5]{x^3-26}\right)^4+\cdots+\left(\sqrt[5]{x^3-26}\right)+1\right]} =$$

$$= \frac{\left[\left(\sqrt[5]{x^3-26}\right)^5-1\right]\left[\left(\sqrt[4]{2x^2-17}\right)^3+\cdots+\left(\sqrt[4]{2x^2-17}\right)+1\right]}{\left[\left(\sqrt[4]{2x^2-17}\right)^4-1\right]\left[\left(\sqrt[5]{x^3-26}\right)^4+\cdots+\left(\sqrt[5]{x^3-26}\right)+1\right]} =$$

$$= \frac{\left(x^3-27\right)\left[\left(\sqrt[4]{2x^2-17}\right)^3+\cdots+\left(\sqrt[4]{2x^2-17}\right)+1\right]}{\left(2x^2-18\right)\left[\left(\sqrt[5]{x^3-26}\right)^4+\cdots+\left(\sqrt[5]{x^3-26}\right)+1\right]} =$$

$$= \frac{\left(x^3-3^3\right)\left[\left(\sqrt[4]{2x^2-17}\right)^3+\cdots+\left(\sqrt[4]{2x^2-17}\right)+1\right]}{2\left(x^2-3^2\right)\left[\left(\sqrt[5]{x^3-26}\right)^4+\cdots+\left(\sqrt[5]{x^3-26}\right)+1\right]} =$$

$\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$
$\alpha^2-\beta^2=(\alpha-\beta)(\alpha+\beta)$

$$= \frac{\cancel{(x-3)}\left(x^2+3x+3^2\right)\left[\left(\sqrt[4]{2x^2-17}\right)^3+\cdots+\left(\sqrt[4]{2x^2-17}\right)+1\right]}{2\cancel{(x-3)}(x+3)\left[\left(\sqrt[5]{x^3-26}\right)^4+\cdots+\left(\sqrt[5]{x^3-26}\right)+1\right]} =$$

$$= \frac{\left(x^2+3x+9\right)\left[\left(\sqrt[4]{2x^2-17}\right)^3+\cdots+\left(\sqrt[4]{2x^2-17}\right)+1\right]}{2(x+3)\left[\left(\sqrt[5]{x^3-26}\right)^4+\cdots+\left(\sqrt[5]{x^3-26}\right)+1\right]}$$

$$\lim_{x\to 3}\frac{\sqrt[5]{x^3-26}-1}{\sqrt[4]{2x^2-17}-1} =$$

$$\lim_{x\to 3}\frac{\left(x^2+3x+9\right)\left[\left(\sqrt[4]{2x^2-17}\right)^3+\cdots+\left(\sqrt[4]{2x^2-17}\right)+1\right]}{2(x+3)\left[\left(\sqrt[5]{x^3-26}\right)^4+\cdots+\left(\sqrt[5]{x^3-26}\right)+1\right]} =$$

$$=\frac{(9+9+9)\left(\underbrace{1+1+\cdots+1}_{4-\varphi o\rho\acute{\varepsilon}\varsigma}\right)}{2(3+3)\left(\underbrace{1+1+\cdots+1}_{5-\varphi o\rho\acute{\varepsilon}\varsigma}\right)}=\frac{3\bullet 9\bullet 4}{2\bullet 2\bullet 3\bullet 5}=\frac{9}{5}$$

4.

Να βρεθεί το όριο $\lim\limits_{x\to 1}\dfrac{x^{\nu+\mu}+\mu x^{\nu}-x^{\mu}-\mu}{x^{\mu+\nu}+\nu x^{\mu}-x^{\nu}-\nu},\ \mu,\nu\in\mathbb{N}^*$

$$\acute{E}\chi\omega:\lim_{x\to 1}\left(x^{\nu+\mu}+\mu x^{\nu}-x^{\mu}-\mu\right)=\lim_{x\to 1}\left(x^{\mu+\nu}+\nu x^{\mu}-x^{\nu}-\nu\right)=0$$

$$\frac{x^{\nu+\mu}+\mu x^{\nu}-x^{\mu}-\mu}{x^{\mu+\nu}+\nu x^{\mu}-x^{\nu}-\nu}=\frac{\underbrace{x^{\nu}\bullet x^{\mu}-x^{\mu}}_{\substack{\text{Βγάζω κοινό}\\ \text{παράγοντα το } x^{\mu}}}+\underbrace{\mu x^{\nu}-\mu}_{\substack{\text{Βγάζω κοινό}\\ \text{παράγοντα το } \mu}}}{\underbrace{x^{\mu}\bullet x^{\nu}-x^{\nu}}_{\substack{\text{Βγάζω κοινό}\\ \text{παράγοντα το } x^{\nu}}}+\underbrace{\nu x^{\mu}-\nu}_{\substack{\text{Βγάζω κοινό}\\ \text{παράγοντα το } \nu}}}=\frac{x^{\mu}\left(x^{\nu}-1\right)x^{\mu}+\mu\left(x^{\nu}-1\right)}{x^{\nu}\left(x^{\mu}-1\right)+\nu\left(x^{\mu}-1\right)}=$$

$$=\frac{\left(x^{\nu}-1\right)\left(x^{\mu}+\mu\right)}{\left(x^{\mu}-1\right)\left(x^{\nu}+\nu\right)}=\frac{\cancel{(x-1)}\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)\left(x^{\mu}+\mu\right)}{\cancel{(x-1)}\left(x^{\mu-1}+x^{\mu-2}+\cdots+x+1\right)\left(x^{\nu}+\nu\right)}=$$

$$=\frac{\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)\left(x^{\mu}+\mu\right)}{\left(x^{\mu-1}+x^{\mu-2}+\cdots+x+1\right)\left(x^{\nu}+\nu\right)}$$

$$\lim_{x\to 1}\frac{x^{\nu+\mu}+\mu x^{\nu}-x^{\mu}-\mu}{x^{\mu+\nu}+\nu x^{\mu}-x^{\nu}-\nu}=\lim_{x\to 1}\frac{\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)\left(x^{\mu}+\mu\right)}{\left(x^{\mu-1}+x^{\mu-2}+\cdots+x+1\right)\left(x^{\nu}+\nu\right)}=$$

$$\frac{\left(\underbrace{1+1\cdots+1}_{\nu-\varphi o\rho\acute{\varepsilon}\varsigma}\right)(1+\mu)}{\left(\underbrace{1+1\cdots+1}_{\mu-\varphi o\rho\acute{\varepsilon}\varsigma}\right)(1+\nu)}=\frac{\mu(1+\mu)}{\nu(1+\nu)}$$

5.

Να βρεθεί το όριο $\lim\limits_{x\to 1}\dfrac{\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa}-1}{\sqrt[4]{x^{2\mu}+x^{\nu}-1}-1},\ \mu,\nu,\kappa\in\mathbb{N}^*$

$$\acute{E}\chi\omega:\lim_{x\to 1}\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa}-1\right]=\lim_{x\to 1}\left[\sqrt[4]{x^{2\mu}+x^{\nu}-1}-1\right]=0$$

$$\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa}-1=\left[\left(x^{2\nu}+x^{\mu}-1\right)-1\right]\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa-1}+\cdots+\left(x^{2\nu}+x^{\mu}-1\right)+1\right]$$

Πολλαπλασιάαζω αριθμητή και παρονομαστή με την παράσταση:

$$\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{2}+\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)+1$$

$$\frac{\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa}-1}{\sqrt[4]{x^{2\mu}+x^{\nu}-1}-1}=$$

$$=\frac{\left[\left(x^{2\nu}-1\right)+\left(x^{\mu}-1\right)\right]\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa-1}+\cdots+\left(x^{2\nu}+x^{\mu}-1\right)+1\right]\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\cdots+1\right]}{\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)-1\right]\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{2}+\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)+1\right]}=$$

$$=\frac{\left[\left(x^{2\nu}-1\right)+\left(x^{\mu}-1\right)\right]\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa-1}+\cdots+\left(x^{2\nu}+x^{\mu}-1\right)+1\right]\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\cdots+1\right]}{\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{4}-1}=$$

$$=\frac{\left[\left(x-1\right)\left(x^{2\nu-1}+\cdots 1\right)+\left(x-1\right)\left(x^{\mu-1}+\cdots+x+1\right)\right]}{\left(x^{2\mu}-1\right)+\left(x^{\nu}-1\right)}\bullet$$

$$\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa-1}+\cdots+\left(x^{2\nu}+x^{\mu}-1\right)+1\right]\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\cdots+\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)+1\right]=$$

$$=\frac{\left(x-1\right)\left[\left(x^{2\nu-1}+x^{2\nu-2}+\cdots+x+1\right)+\left(x^{\mu-1}+x^{\mu-2}\cdots+x+1\right)\right]}{\left(x-1\right)\left(x^{2\mu-1}+x^{2\mu-2}\cdots+x+1\right)+\left(x-1\right)\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)}\bullet$$

$$\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa-1}+\cdots+\left(x^{2\nu}+x^{\mu}-1\right)+1\right]\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\cdots+\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)+1\right]=$$

$$=\frac{\cancel{\left(x-1\right)}\left[\left(x^{2\nu-1}+x^{2\nu-2}+\cdots+x+1\right)+\left(x^{\mu-1}+x^{\mu-2}\cdots+x+1\right)\right]}{\cancel{\left(x-1\right)}\left[\left(x^{2\mu-1}+x^{2\mu-2}\cdots+x+1\right)+\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)\right]}\bullet$$

$$\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa-1}+\cdots+\left(x^{2\nu}+x^{\mu}-1\right)+1\right]\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\cdots+\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)+1\right]=$$

$$=\frac{\left[\left(x^{2\nu-1}+\cdots+x+1\right)+\left(x^{\mu-1}+\cdots+x+1\right)\right]\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa-1}+\cdots+1\right]\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\cdots+1\right]}{\left(x^{2\mu-1}+x^{2\mu-2}\cdots+x+1\right)+\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)}$$

$$\lim_{x\to 1}\frac{\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa}-1}{\sqrt[4]{x^{2\mu}+x^{\nu}-1}-1}=$$

$$\lim_{x\to 1}\frac{\left[\left(x^{2\nu-1}+\cdots+1\right)+\left(x^{\mu-1}+\cdots+1\right)\right]\left[\left(x^{2\nu}+x^{\mu}-1\right)^{\kappa-1}+\cdots+1\right]\left[\left(\sqrt[4]{x^{2\mu}+x^{\nu}-1}\right)^{3}+\cdots+1\right]}{\left(x^{2\mu-1}+x^{2\mu-2}\cdots+x+1\right)+\left(x^{\nu-1}+x^{\nu-2}+\cdots+x+1\right)}=$$

$$=\frac{\left[\left(\underbrace{1+1+\cdots+1}_{2\nu-\varphi o\rho\acute{\varepsilon}\varsigma}\right)+\left(\underbrace{1+1+\cdots+1}_{\mu-\varphi o\rho\acute{\varepsilon}\varsigma}\right)\right]\left(\underbrace{1+1+\cdots+1}_{\kappa-\varphi o\rho\acute{\varepsilon}\varsigma}\right)(1+1+1+1)}{\left(\underbrace{1+1+\cdots+1}_{2\mu-\varphi o\rho\acute{\varepsilon}\varsigma}\right)+\left(\underbrace{1+1+\cdots+1}_{\nu-\varphi o\rho\acute{\varepsilon}\varsigma}\right)}=\frac{4\kappa(2\nu+\mu)}{2\mu+\nu}$$

ΑΣΚΗΣΕΙΣ

1.

$$N\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \acute{o}\rho\iota o \lim_{x\to 0}\frac{\left(5x^3+2x+1\right)^8-1}{\left(3x^2+4x+1\right)^5-1}$$

2.

$$N\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \acute{o}\rho\iota o \lim_{x\to 3}\frac{\left(x^3-26\right)^7-1}{\sqrt[5]{x^2-8}-1}$$

3.

$$N\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \acute{o}\rho\iota o \lim_{x\to 1}\frac{\sqrt[5]{2x^3-1}-1}{\sqrt[4]{5x^2-4}-1}$$

4.

$$N\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \acute{o}\rho\iota o \lim_{x\to 1}\frac{x^{\nu-\mu}+\mu x^{\nu}-x^{-\mu}-\mu}{x^{\mu-\nu}+\nu x^{\mu}-x^{-\nu}-\nu},\mu,\nu\in\mathbb{N}^*$$

5.

$$N\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \acute{o}\rho\iota o \lim_{x\to 1}\frac{\left(x^{2\mu}+x^{\nu}-1\right)^4-1}{\sqrt[4]{x^{2\nu}+x^{\mu}-1}-1},\mu,\nu\in\mathbb{N}^*$$

6.

$$N\alpha\ \beta\rho\varepsilon\theta\varepsilon\acute{\iota}\ \tau o\ \acute{o}\rho\iota o \lim_{x\to 1}\frac{x^{\nu}+x^{\nu-1}+\cdots+x^2+x-\nu}{x-1},\nu\in\mathbb{N}^*$$

$$\Delta\acute{\iota}\nu\varepsilon\tau\alpha\iota: 1+2+\cdots+\nu=\frac{\nu(\nu+1)}{2}$$

$$\Upsilon\pi\acute{o}\delta\varepsilon\iota\xi\eta: x^{\nu}+x^{\nu-1}+\cdots+x^2+x-\nu \overset{\nu=\underbrace{1+1+\cdots+1}_{\nu-\varphi o\rho\acute{\varepsilon}\varsigma}}{=} x^{\nu}+x^{\nu-1}+\cdots+x^2+x-\left(\underbrace{1+1+\cdots+1}_{\nu-\varphi o\rho\acute{\varepsilon}\varsigma}\right)=$$

$$\left(x^{\nu}-1\right)+\left(x^{\nu-1}-1\right)+\cdots+\left(x^3-1\right)+\left(x^2-1\right)+(x-1)=$$

$$(x-1)\left(x^{\nu-1}+\cdots+1\right)+(x-1)\left(x^{\nu-2}+\cdots+1\right)+\cdots+(x-1)\left(x^2+x+1\right)+(x-1)(x+1)+(x-1)=$$

$$(x-1)\left(\underbrace{x^{\nu-1}}_{1-\varphi o\rho\acute{\alpha}}+\underbrace{x^{\nu-2}+x^{\nu-2}}_{2-\varphi o\rho\acute{\varepsilon}\varsigma}+\cdots+\underbrace{x^2+\cdots+x^2}_{(\nu-2)-\varphi o\rho\acute{\varepsilon}\varsigma}+\underbrace{x+\cdots+x}_{(\nu-1)-\varphi o\rho\acute{\varepsilon}\varsigma}+\underbrace{1+\cdots+1}_{\nu-\varphi o\rho\acute{\varepsilon}\varsigma}\right)=$$

$$(x-1)\left[x^{\nu-1}+2x^{\nu-2}+\cdots+(\nu-2)x^2+(\nu-1)x+\nu\right]$$